Landauer-Büttiker approximation

(i) Consider the mesoscopic system with phase coherence (its characteristic length is less than the phase coherence length l_ϕ) shown in figure 1. Leads (contacts)

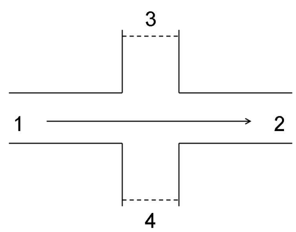

Figure 1. Schematic representation of a 4-terminal coherent mesoscopic system.

1 and 2 are the source and the drain (are used as current leads, with the electric current introduced through 1 and extracted from 2), while leads 3 and 4 are voltage leads. The system is symmetric and conductances are given by $G_{\alpha\beta} = G_0,\ G_{\alpha\gamma} = G_1,\ G_{\gamma\delta} = G_2$, where $\alpha,\ \beta$ are 1 or 2 and $\gamma,\ \delta$ are 3 or 4. Show that $V_1 + V_2 = V_3 + V_4$.

(ii) Taking into account the sum rule (what is its origin?):

$$G_{12} + G_{13} + G_{14} = \frac{2e^2}{h} N_c, \tag{1}$$

where N_c is the number of channels (transverse subbands with propagating states at Fermi energy), calculate electrical resistance measured between leads (contacts) 1 and 2, $R_{12,12}$.

(iii) Discuss the result you have obtained above in the limits $G_0 << \frac{2e^2}{h} N_c$ and, respectively, $G_0 = \frac{2e^2}{h} N_c$. Explain the physics corresponding to those limits.

(iv) In the following you will consider the system shown in figure 2, consisting of N identical scattering regions connected by $N-1$ voltage leads (external contacts connected to voltmeters). The transmission function (transmission probability) through each individual scattering region is T, scattered electrons entering voltage terminals with probability 1 (in other words, voltage terminals are perfectly absorbant - such contacts are known as invasive contacts). Show that:

(a) The voltage measured between two successive terminals is:

$$V_{n+1} - V_n = \frac{V}{N},\ n = 0, 1, \cdots, N-1. \tag{2}$$

(b) The electrical resistance of the whole system is:

$$R = \frac{V}{I} = \frac{h}{2e^2} \frac{N}{T}. \tag{3}$$

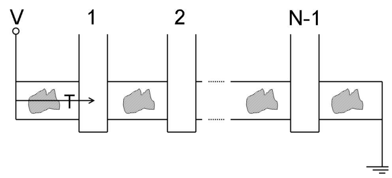

Figure 2. Schematic representation of a physical system consisting of N scattering regions with phase coherence, connected by voltage terminals.

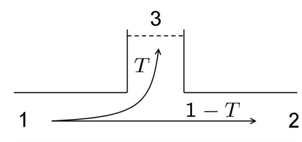

Figure 3. Schematic representation of a coherent mesoscopic system with 3 leads.

The result in eq. (3) may be rewritten as:

$$R = \frac{h}{2e^2}\left(\frac{1-T}{T} + (N-1) + 1\right). \tag{4}$$

Explain the physical origin of the terms in parentheses.

(v) Consider the 3-terminal coherent system shown in figure 3. Leads 1 and 2 are current terminals (source and drain), while lead 3 is a voltage terminal. The following conducatnce values are known $G_{12} = G_{21} = \frac{2e^2}{h}(1-T)$, $G_{13} = G_{31} = \frac{2e^2}{h}T$. The parameter T characterizes the coupling of transport channels to the voltage terminal. Electrical contacts are perfect absorbant (no reflection of electrons entering particle reservoirs attached to them). Calculate the electrical resistance $R_{12,12}$. Comment your result.